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MISCONCEPTION AND DEVELOPMENT OF CONCEPTUAL REASONING OF PROSPECTIVE TEACHER STUDENTS TOWARDS THE NON-COMMUTATIVE PROPERTY OF COMPOSITION FUNCTIONS THROUGH PEER TEACHING-BASED DISCOVERY LEARNING

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ABSTRACT

Conceptual understanding of function composition, particularly its non commutative property, is a challenge for prospective mathematics teachers. This difficulty is often indicated by the emergence of miscoceptions stemming from inappropriate generalizations of arithmetic operations of intuitive judgments of results without a clear relational basis. This study aims to describe the initial misconceptions and the development of prospective teachers' conceptual reasoning regarding the non commutative property of function composition through the implementation of contextualized discovery learning within a peer teaching framework. The study employed a qualitative approach with a descriptive-exploratory design. The research participants were graduate students enrolled in a masters program in mathematics education who were directly involved in peer teaching activities. Data were collected through video recordings of instructional practices, student worksheets, prediction sticky notes, and field notes. Data analysis was conducted using qualitative procedures involving data reduction, thematic coding, and interpretative analysis. The findings indicate that, at the initial stage, students tended to exhibit misconceptions based on cross concept analogies and intuitive reasoning that had not yet developed into conceptual understanding. Through presentation and discussion activities in the discovery learning process, students demonstrated a shift in reasoning from a focus on final results toward relational understanding related to the order of function mappings. These findings suggest that discovery learning supported by peer teaching facilitates cognitive conflict and meaning negotiation, thus contributing to the development of prospective mathematics teachers conceptual understanding of the non commutative property of function composition.

Keywords: Conceptual Reasoning, Discovery Learning, Function Composition, Misconceptions, Peer Teaching

INTRODUCTION

Conceptual understanding is an essential goal in mathematics learning because it forms the foundation for reasoning, communication, and problem-solving skills. The independent curriculum emphasizes that mathematics learning should encourage students to understand concepts meaningfully, not simply master procedures and formulas (Ministry of Education, Culture, Research, and Technology of the Republic of Indonesia, 2022). In line with this, the NCTM emphasizes the importance of learning that fosters conceptual and relational understanding so that students can explain the mathematical reasoning behind a procedure or result (National Council of Teachers of Mathematics, 2020).

However, various studies show that conceptual understanding of mathematics remains a challenge, including for prospective teacher students. This difficulty is clearly evident in the material on compositional functions, which requires relational thinking skills and an understanding of symbolic meaning. Research by Hesti and Erita (2023) shows that students experience errors and misconceptions in solving compositional function problems, both at the procedural and conceptual levels, including in understanding the basic properties of functions.

One common misconception relates to the commutative nature of composition functions. Students tend to generalize the properties of arithmetic operations, such as addition and multiplication, to the context of composition functions without considering the differences in meaning between numerical operations and function mappings. As a result, students may provide answers based on intuitive results or incorrect analogies, without adequate conceptual reasoning. This finding suggests that correct answers do not necessarily reflect a complete understanding of the concept (Hesti & Erita, 2023; Yohanes et al., 2022).

A learning approach oriented toward students' activities and thought processes is considered relevant for uncovering and reconstructing these misconceptions. Discovery learning provides space for students to construct knowledge through exploration, discussion, and independent drawing of conclusions. Several studies have shown that discovery learning, especially when combined with a contextual approach, can support the development of conceptual understanding, mathematical communication, and critical thinking skills (Halawa & Harefa, nd; Shaqila & Zetriuslita, 2023; Kewa et al., 2025). More generally, active learning has been shown to have a positive impact on student engagement and performance in science and mathematics learning (Freeman et al., 2014).

Discovery learning becomes even more meaningful when integrated with peer teaching. Through peer teaching, students engage in discussion, explanation, and reflection together, enabling the exchange of ideas and negotiation of meaning. Research by Topping (2005) shows that peer teaching positively contributes to the learning process and the development of teaching skills, particularly for student teachers.

However, most research related to discovery learning and peer teaching still focuses on learning outcomes or the improvement of specific skills. Studies specifically examining how initial misconceptions emerge, develop, and are negotiated during the learning process, particularly through the analysis of video-based learning practices, are still relatively limited. Yet, analyzing learning videos has great potential to reveal the dynamics of interactions and students' thinking processes in greater depth (Gaudian & Chalie, 2015; Sherin & Van Es, 2009).

Based on the description, this study aims to describe the initial misconceptions and the development of conceptual reasoning of prospective teacher students regarding the non-commutative nature of composition functions through the implementation of discovery learning with a contextual approach in a peer teaching scheme.

RESEARCH METHODS

This study used a qualitative approach with a descriptive-exploratory design. This approach was chosen because the study focused on examining the learning process, specifically how misconceptions related to the non-commutative nature of composition functions emerged, developed, and negotiated during the implementation of discovery learning with a contextual approach in a peer teaching scheme.

The research subjects were students of the Master of Mathematics Education Study Program who were taking the Innovative Mathematics Learning Study course. Subject selection was conducted using purposive sampling with the following criteria: (1) direct involvement in peer teaching activities that implemented discovery learning with a contextual approach, and (2) availability of adequate video documentation of learning practices for analysis. In the learning context, students acted as both peer tutors and learning participants.

Research data was collected through video recordings of peer teaching practices as the primary data source, supported by Student Worksheets (LKPD), initial student predictions made using sticky notes, and field notes. Video recordings were used to observe the stages of discovery learning, the dynamics of discussion interactions, and the emergence and facilitation of misconceptions during the learning process.

In this study, the researcher acted as a non-participant observer, not directly involved in the learning process. To minimize the potential for subjective bias, data analysis was conducted in stages and discussed through peer discussions, supported by triangulation of data sources between video recordings, student worksheets, and field notes.

The unit of analysis in this study is the conceptual responses of prospective teacher students regarding the non-commutative nature of composition functions, which emerged during the discovery learning stage. These conceptual responses include initial predictions, oral explanations during presentations and discussions, and arguments put forward in response to intergroup questions. The analysis focused on the forms of misconceptions, the shift in reasoning from intuitive results to relational understanding, and the facilitation strategies that emerged in peer teaching interactions.

Data analysis was conducted qualitatively through the stages of data reduction, thematic coding, interpretive analysis, and conclusion drawing. Coding focused on the stages of discovery learning, the types of misconceptions that emerged, patterns of conceptual reasoning development, and the dynamics of student discussions. Data validity was maintained through source triangulation and peer discussions to ensure consistency of interpretation.

RESULTS AND DISCUSSION

In the stimulation stage, students were asked to predict whether function composition is commutative by writing on sticky notes. The results showed that most students predicted that function composition is commutative. One reason that emerged was that addition and multiplication operations apply the commutative property, so students assumed that function composition has the same property. This response indicates a misconception based on cross-conceptual analogy, namely the tendency of students to transfer the properties of arithmetic operations into the context of function operations without considering the differences in the objects and meanings of the operations. In this case, the composition symbol is perceived as a numerical operation, not as a function mapping.

On the other hand, some students predicted that function composition is not commutative, arguing that the results would inevitably be different. While this conclusion aligns with the general nature of function composition, the reasoning provided was general and lacked an explanation of the order of function mappings or the meaning of the composition symbol. This type of response reflects an intuitive, yet preconceptual understanding, where students are able to guess the final result but lack a sufficient foundation for mathematical reasoning. This finding confirms that correct answers do not always represent a complete conceptual understanding.

The diversity of responses at this early stage indicates that students' understanding of the commutative nature of composition functions falls on a spectrum, ranging from mistaken generalizations based on arithmetic analogies to intuitions about results without a clear relational basis. This condition is the starting point for cognitive conflict, which is crucial in the discovery learning process.

During the presentation and discussion phase of the investigation results, students began to demonstrate a growing understanding by linking the differences in composition results to the order of function mapping. Several students stated that composition results can differ because the order of functions affects the results, leading them to conclude that not all function compositions are commutative. This statement marks a shift from analogy- and intuition-based thinking to a rational understanding. Although the explanations provided are still not entirely logically consistent and have not been formulated in systematic mathematical reasoning.

Intergroup discussions led to confusion when groups obtained identical composition results, prompting questions about the requirements for a composition function to be commutative. This question indicates that students are beginning to think conditionally, no longer viewing concepts as right or wrong. At this stage, cognitive conflict no longer arises from differences in initial predictions but rather from differences in empirical results between groups, which demand deeper conceptual explanations.

This process demonstrates the role of discovery learning in facilitating the negotiation of meaning through discussion and presentation of results. Students are not only guided to find answers but also to gradually revise their reasoning and reasoning structures. Thus, learning doesn't stop at correcting answers but rather encourages a shift in thinking from simply the end result to a relational understanding of the sequence of function mappings.

The implications of these findings suggest that discovery learning based on group discussions and presentations can be an effective way to uncover and reconstruct students' misconceptions. This type of learning allows students to gradually revise initial misconceptions and develop a deeper conceptual understanding of the properties of function composition, particularly regarding the non-commutative property and the special consistency conditions that accompany it.

CONCLUSION

Based on the results and discussion, it can be concluded that student teachers still show initial misconceptions regarding the commutative nature of composition functions, which generally originate from generalizations of the properties of arithmetic operations and intuition of results without adequate relational understanding. Through the implementation of discovery learning with a contextual approach in a peer teaching scheme, there is a development in students' conceptual reasoning marked by a shift in focus from the final result to an understanding of the order of function mapping.

Group discussions and presentations play a crucial role in eliciting cognitive conflict and fostering negotiation of meaning among students. Learning not only results in corrected answers but also in gradual improvements in mathematical reasoning and reasoning structures. Thus, peer-teaching-based discovery learning has the potential to be an effective approach for uncovering and reconstructing pre-service teachers' conceptual misconceptions on compositional functions, particularly regarding the non-commutative property.

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